An Optimal Mechanism for Competitive Markets with Adverse Selection

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Abstract

We construct an optimal mechanism for competitive markets with adverse selection. In the mechanism, a firm is asked to serve as the market maker. The market maker offers a menu of products (henceforth the minimum standards). Rivals compete by offering any other product they wish but they always need to include in their offers these minimum standards. We show that this endogenous form of restriction in the set of products firms are allowed to offer, reduces the set of profitable deviations to the point that an equilibrium exists in every market with adverse selection. Furthermore, we characterise general, well-studied environments, in which the set of equilibrium indirect utility profiles coincides with the set of efficient utility profiles.

JEL CLASSIFICATION: D02, D82, D86

KEYWORDS: Adverse Selection, Competition, Optimal Mechanism, Existence, Efficiency.

I. INTRODUCTION

In their seminal contribution, Rothschild and Stiglitz (1976) (henceforth RS) analysed a competitive market with adverse selection and argued that if firms, e.g. the uninformed parties in the market, unrestrictedly compete by offering products, e.g. insurance contracts, an equilibrium may fail to exist. In order to reduce the set of profitable deviations and guarantee existence, Wilson (1977) restricted the set of products firms are allowed to offer. Even though this was not explicitly modelled in an extensive form game, Wilson (1977) argued that these restrictions should be thought as the reduced form of a dynamic process in which firms offer products that can be later withdrawn if they become loss-making. Under these circumstances, he showed that an equilibrium exists.[1] Miyazaki (1977) and Spence (1978) extended the idea of Wilson (1977) by allowing firms to offer menus of products, instead of single products, and found that equilibrium not only exists but is also efficient. Yet, a later contribution by Hellwig (1987) highligthed that the set of equilibrium

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[1] Riley (1979) proposed a related dynamic process according to which firms can add products as a response to products offered by other firms.

allocations might not be this envisioned by Wilson (1977), Miyazaki (1977) and Spence (1978), if these dynamics are modelled in an extensive form game.[2] On the contrary, a plethora of inefficient allocations can be sustained as equilibria. Ever since, a fundamental question remains open: Is there a mechanism, i.e. game form, that restricts the way firms compete in order to guarantee existence of equilibrium without distorting efficiency?

In this paper, we propose such a novel mechanism. Our mechanism works as follows: A firm (henceforth the market maker) is randomly selected to select a menu of products (henceforth the minimum standards). This menu then needs to be offered by all other firms in the market who decide to stay active. In addition to these minimum standards, firms can offer any other products they wish. In that sense, restrictions in the offered contracts are not imposed exogenously but endogenously by a player in the game. A regulator only needs to ensure that these restrictions are indeed not violated.

We establish two main results. First, as expected, we show that restrictions in the product space reduce the set of profitable deviations to the point that an equilibrium always exists. We formally prove the result by characterising equilibrium strategies and demand functions, all in accordance with subgame perfection. The main intuition behind this universal existence result is that if the market maker selects an efficient menu of products, the rest of the firms are restricted to include this in their offered menus of products and hence are unable to deviate and try to selectively attract high profit types. Second, we characterise a wide class of environments, that include almost all well-studied environments, in which non-efficient outcomes cannot be sustained in equilibrium. The idea behind this result is almost identical to the idea behind Bertrand competition: Firms have always an incentive to deviate from a non-efficient outcome and undercut rivals in order to increase their profits. Sufficient conditions therefore are specified to make sure that this undercutting is always possible.

Our mechanism is not unique. In fact, related mechanisms have been proposed in the literature. For instance, Asheim and Nilssen (1996) allow firms to renegotiate the products with their customers, but prohibit them to discriminate among the different types in the renegotiation stage. Diasakos and Koufopoulos (2011), extend the game of Hellwig (1987) by allowing firms to commit to the menus of products they offer and they show that equilibrium exists and is efficient. In Netzer and Scheuer (2014), firms offer menus of products and decide whether to stay in the market or not after they observe the menus of products of rivals. To become inactive, they have to pay an exogenously-given withdrawal cost. Existence of equilibrium crucially depends on the exogenous specification of the withdrawal cost. If the cost is high, an equilibrium fails to exist for the same reason it fails to exist in RS. For certain (small) values of the withdrawal cost equilibrium exists and coincides and is efficient. In a similar vein, Mimra and Wambach (2011) allow firms, instead of becoming inactive, to withdraw individual products from those they have offered in an

[2] Hellwig (1987) followed the tradition of RS and analysed an environment in which firms are only allowed to offer one product. Recently, Mimra and Wambach (2011), Netzer and Scheuer (2014) allowed firms to offer menus of products and found that the set of equilibrium allocation is even larger than this in Hellwig (1987). The idea is that f firms are allowed to offer products that can be later withdrawn, some sort of folk theorem prevails.

endogenously ending number of rounds. They show that, without further restrictions, the equilibrium set of this game contains every incentive compatible and positive profit allocation. They then examine which of these equilibria are robust to entry and they show that only an efficient allocation indeed survives. Lastly, Picard (2014) allows firms to offer "participating products" such that a consumer who buys a product needs to "participate" in the profits of the firm who offered it. Under this modification, he shows that an equilibrium exists and is efficient.[3]

We believe that there are at least two significant differences between our mechanism and those mentioned before. First and foremost, our mechanism applies to a broader class of environments. Specifically, as we show, for our existence result, no particular conditions are necessary, and, for our efficiency result, the sufficient conditions are considerably milder than these satisfied by the insurance environment of RS. On the contrary, all the aforementioned papers are taken place in the, rather restrictive, environment of RS. Second, our mechanism is relatively simpler and only slightly departs from traditional product competition.

The rest of the paper is organised as follows. In Section II we describe the general environment as well as special environments. In Section III, we describe the mechanism. In Section IV, we state and prove the results. First we show that in every selection market an equilibrium exists. The proof is incorporated in the main text in order to provide intuition. Second, we show that in special well-known environments where utility and cost functions are strictly increasing, concave and additively separable, every equilibrium is efficient. In Section IV, we discuss several points that need further clarification.

II. The Model

■ **The General Environment.** There is a measure one of consumers. Each consumer belongs to a certain class (type). We assume that the set of types is finite and denoted by Θ with a representative element $\theta \in \Theta$. The restriction to finite sets is not essential but mostly practical and to avoid measurability problems. All our results extend, non-trivially and under technical difficulties, to uncountably infinite sets. The type of a consumer may include any non-observable characteristic such as riskiness, attitude towards risk, income etc, or observable characteristics such as gender, pre-existing conditions etc that cannot be used for discrimination. The share of type θ consumers in the population is λ^θ with $\sum_{\theta\in\Theta} \lambda^\theta = 1$

There are three active symmetric firms in the market $\{0, 1, 2\}$. The restriction to only three firms simplifies notation, analysis and proof, even though all our results hold as long as there are at least two active firms. We extensively discuss that point when in the robustness section.

Firms supply products, e.g. financial products, insurance contracts etc, in the market. A product is denoted by $x \in X$. The definition of a product in our environment includes its technical characteristics, e.g. the insurance coverage in case of insurance, as well as its price, e.g. the premium in case of insurance. A consumer of type θ has utility function $U^\theta : X \to \mathbb{R}$. The status quo

[3]Bisin and Gottardi (2006), and Citanna and Siconolfi (2013) are also related in terms of their intention, even though in a Walrasian environment.

product of any consumer is x_0 and the status quo utility of type θ is $\underline{U}^\theta$. This may correspond for example to either no insurance at all or some elementary insurance perhaps provided by the state.

Every product is associated with a (net) profit. The profit function for type θ is $\zeta^\theta : X \to \mathbb{R}$. On top of any financial cost, firms also incur physical costs. These costs may be in terms of labour and physical capital required to create and accommodate a product. In almost all studies of markets with adverse selection these costs are usually set to zero. This is mainly for two reasons. First, if firms face non-constant returns to scale, the analysis of imperfect competition becomes particularly tedious.[4] Second, incorporating physical costs may prevent the researcher from focusing on the effect of adverse selection in the market outcome.

An allocation $\mathbf{x} = (x^\theta)_{\theta\in\Theta}$ is an unordered set of products indexed by the set of types. An allocation $\mathbf{x} = (x^\theta)_{\theta\in\Theta}$ is incentive compatible if and only if for all θ, $x^\theta \in \arg\max_{\theta'\in\Theta} U^\theta(x^{\theta'})$. An incentive compatible allocation is individually rational if and only if $U^\theta(x^\theta) \geq \underline{U}^\theta$ for every $\theta \in \Theta$. The set of incentive compatible and individually rational allocations is denoted as $\mathbb{X}^{IC}$. The (net) profit of allocation $\mathbf{x}$ is: $Z(\mathbf{x}) = \sum_{\theta\in\Theta} \lambda^\theta \zeta^\theta(x^\theta)$. An allocation $\mathbf{x}$ is efficient if and only if: (i) it is incentive compatible, individually rational and positive profit and (ii) there exists no other incentive compatible, individually rational and and positive-profit allocation $\tilde{\mathbf{x}}$ such that for each $\theta \in \Theta$, $U^\theta(\tilde{x}^\theta) \geq U^\theta(x^\theta)$ with the inequality being strict for at least one $\theta \in \Theta$. A utility profile is $\mathbf{U} = (U^\theta(x^\theta))_{\theta\in\Theta}$. The set of efficient utility profiles is denoted as $\mathcal{U}^E$.

□ **Special Environments.** Various special cases of the above model have been studied in the literature. In what follows we provide a large set of environments that include most well-studied environments.

ASSUMPTION A1: $X = \mathbb{R}^{|\Omega|}$, $|\Omega| \geq 2$, and for every θ, $U^\theta(x)$, $\zeta^\theta(x)$ are additively separable with

$$U^\theta(x) = \sum_{\omega=1}^{\Omega} p^\theta_\omega v_\omega(x_\omega)$$

$$\zeta^\theta(x) = \sum_{\omega=1}^{\Omega} p^\theta_\omega \phi_\omega(x_\omega)$$

where $v_\omega(\cdot)$, $\phi_\omega(\cdot)$ are strictly increasing and concave for every $\omega = 1, ..., \Omega$ and v_ω is strictly concave for at least one $\omega = 1, ..., \Omega$.

1. *The Insurance Market.* $|\Omega|$ is the number of possible individual states, with $\omega = 1$ the state of no accident and $\omega = 2, ..., \Omega$ the states where a consumer faces an accident. Consumer of type θ starts with initial wealth W^θ and can suffer state-dependent losses $\ell_\omega \geq 0$, with $\ell_1 = 0$ and $\ell_\omega < \ell_{\omega+1}$ for every $\omega = 2, ..., \Omega$. The space of insurance contracts is $X = \mathbb{R} \times \mathbb{R}_+^{|\Omega|-1}$. In other words

[4] Recall the case of Bertrand competition in a homogeneous product market where firms face decreasing returns to scale.

a contract specifies a premium x_1 and reimburses the consumer in case he has an accident.[5] The coefficients are restricted to represent probabilities, or $p^\theta_\omega \geq 0$ and $\sum_{\omega=1}^{\Omega} p^\theta_\omega = 1$. The expected utility of type θ from insurance contract x is given by $U^\theta(x) = \sum_{\omega=1}^{\Omega} p^\theta_\omega v_\omega(W^\theta - \ell_\omega + x_\omega)$, where $v_\omega(\cdot)$ is strictly concave for every ω. The (expected) financial profit of contract x is $\zeta^\theta(x) = \sum_{\omega=1}^{\Omega} p^\theta_\omega x_\omega$. The status quo utility of type θ is $\underline{U}^\theta = \sum_{\omega=1}^{\Omega} p^\theta_\omega v_\omega(W^\theta - \ell_\omega)$.

2. *The Rothschild-Stiglitz-Wilson Market.* If $\Omega = 2$ and $W^\theta = W$ for every θ, the model is that of RS and Wilson (1977).[6] Each type starts with the same level of wealth but different types differ with respect to their riskiness. Higher types are less risky than lower types.

3. *The Quasi-linear Market.* If wealth effects are relatively negligible, then each consumer is mostly interested in maximising the difference between the value from a product and the price paid. Then we are in a quasi-linear environment, e.g. as it is quite common in most studies of industrial organisation. The utility of type θ is $U^\theta(x) = -x_1 + \sum_{\omega=2}^{\Omega} p^\theta_\omega v_\omega(x_\omega)$ from making transfer x_1 and receiving transfers x_ω for $\omega = 2, ..., \Omega$. The profit of product x is $\zeta^\theta(x) = x_1 - \sum_{\omega=2}^{\Omega} p^\theta_\omega \phi_\omega(x_\omega)$. $v_\omega(\cdot)$ and $\phi_\omega(\cdot)$ are strictly concave for every ω.

III. The Mechanism

■ **Description of the Mechanism.** Firms compete by offering menus of products. The set of menus of products from which firms make choices is $\mathbb{A}$, i.e. the set of all finite subsets of X. We assume that $x_0 \in \alpha$ for every $\alpha \in \mathbb{A}$. We further assume that the market maker is always firm 0.[7] The market maker moves first and selects a menu of products α_0 (the minimum standards). Each of the two other firms $i = 1, 2$ selects a new menu of products after having observed the choice of the market maker. Alternatively, firm $i = 1, 2$ can select to stay out of the market if it wishes. Formally, $\alpha_i : \mathbb{A} \to \mathbb{A} \cup \{\emptyset\}$ for every $i = 1, 2$. As we have already explained verbally the effective set of products supplied by firm $i = 1, 2$, if this decides to stay in the market, expands to $\alpha_0 \cup \alpha_i$. A profile of menus of products is $\boldsymbol{\alpha} = (\alpha_0, \alpha_1, \alpha_2)$. The demand function of type θ when he faces a set of products $\gamma \subset X$ is $\xi^\theta(\gamma) = \arg\max_{x\in\gamma} \ U^\theta(x)$ and the indirect utility is $V^\theta(\gamma) = \max_{x\in\gamma} \ U^\theta(x)$. An indirect utility profile is $\mathbf{V}(\gamma) = (V^\theta(\gamma))_{\theta\in\Theta}$.

□ **Definitions.** In order to define profits, we first need to define (aggregate) firm demand functions. Let $Q : \Theta \times X \times \mathbb{A}^3 \to R_+$, and $\boldsymbol{Q} = (Q_0, Q_1, Q_2)$.

Definition (Demand Profile:) **Q** is a profile of (aggregate) demand functions if and only if

[5] To be clear accident here means any contingency in which an individual faces a financial loss. If we consider health insurance, then an accident can be anything from a visit to the doctor to hospitalisation. If we consider automobile insurance, then accident is anything related to a material or medical accident.

[6] Wilson (1977) studied a model where $|\Theta|$ is any finite number. RS concentrated on $|\Theta| = 2$. In this model, there are only two individual states, no accident and accident respectively and two types, a high-risk and a low-risk type.

[7] The results remain unchanged even if the market maker is randomly selected and can be any of the three firms. See Robustness on Discussion.

$\forall \quad \theta, \quad \boldsymbol{\alpha}$:

- $Q_0^\theta(x, \boldsymbol{\alpha}) = Q_1^\theta(x, \boldsymbol{\alpha}) = Q_2^\theta(x, \boldsymbol{\alpha}) = 0 \quad \text{if} \quad x \neg \in \alpha_0 \cup \alpha_1 \cup \alpha_2$
- $Q_0^\theta(x, \boldsymbol{\alpha}) = 0, \quad \text{if} \quad x \neg \in \alpha_0 \cap \xi^\theta(\alpha_0 \cup \alpha_1 \cup \alpha_2)$
- $Q_i^\theta(x, \boldsymbol{\alpha}) = 0, \quad \text{if} \quad x \neg \in (\alpha_0 \cup \alpha_i) \cap \xi^\theta(\alpha_0 \cup \alpha_1 \cup \alpha_2), \quad \forall \quad i = 1, 2$
- $\sum_{x \in \alpha_0 \cap \xi^\theta(\alpha_0 \cup \alpha_1 \cup \alpha_2)} Q_0^\theta(x, \boldsymbol{\alpha}) + \sum_{i=1}^2 \sum_{x \in (\alpha_0 \cup \alpha_i) \cap \xi^\theta(\alpha_0 \cup \alpha_1 \cup \alpha_2)} Q_i^\theta(x, \boldsymbol{\alpha}) = \lambda^\theta$

In words, $\mathbf{Q}$ is a profile of (aggregate) demand functions if and only the measure of type θ consumers who purchase product x from firm i is zero when: (i) This product does not belong to the set of products that are available in the market, (ii) It belongs to the set of products that are available in the market, but does not belong to the set of optimal choices for type θ. The last condition, states that the sum of the demands for the optimal choices for type θ equals the ex ante measure of type θ. Note that for a given profile of menus of products, a demand profile may be indeterminate. To see this consider the scenario where for a profile of menus of products there are two products that belong to the set of optimal choices for type θ. There may be two different demand profiles that satisfy the rationality and feasibility conditions characterised by different allocations of the same type in the two different products. Therefore, there may be more than one function that satisfy the definition.

The expected profits of the market maker and firm $i = 1, 2$ are given by the sum of the weighted sum of profits given an aggregate demand profile:

$$\Pi_0(\alpha_0, \boldsymbol{\alpha}_{-0} | Q_0) = \begin{cases} \sum_{\theta \in \Theta} \sum_{x \in \alpha_0} \zeta^\theta(x) Q_0^\theta(x, \boldsymbol{\alpha}), & \text{if } \alpha_0 \neq \emptyset \\ 0, & \text{if } \alpha_0 = \emptyset \end{cases}$$

$$\Pi_i(\alpha_0, \alpha_i, \alpha_{-i} | Q_i) = \begin{cases} \sum_{\theta \in \Theta} \sum_{x \in \alpha_0 \cup \alpha_i} \zeta^\theta(x) Q_i^\theta(x, \boldsymbol{\alpha}), & \text{if } \alpha_i \neq \emptyset \\ 0, & \text{if } \alpha_i = \emptyset \end{cases}$$

We can now define a (subgame perfect) Nash equilibrium. The endogenous variables are the profile of menus of products selected by firms and the aggregate demand profile.

DEFINITION (Equilibrium:) A subgame perfect Nash equilibrium is a profile of menus of products and demands $(\bar{\boldsymbol{\alpha}}, \bar{\boldsymbol{Q}})$ such that:

(i) $\bar{\alpha}_0 \in \underset{\alpha_0 \in \mathbb{A} \cup \{\emptyset\}}{\arg\max} \quad \Pi_0(\alpha_0, \bar{\boldsymbol{\alpha}}_{-0} | \bar{Q}_0)$

(ii) For every $i = 1, 2$ and α_0, $\bar{\alpha}_i \in \underset{\alpha_i \in \mathbb{A} \cup \{\emptyset\}}{\arg\max} \quad \Pi_i(\alpha_0, \alpha_i, \bar{\alpha}_{-i} | \bar{Q}_i)$

According to this definition an equilibrium consists of a profile of menus of products and demands such that: (1) The menu of firm 0 is optimal given the menus (strategies) of firms $i = 1, 2$ and the demand function of firm 0, (2) For every possible action of firm 0 (even off-the-equilibrium), the strategy of firm i is optimal given the strategy of firm 0 and $-i$ and the demand function of firm i.[8] Subgame perfection is embedded by requiring the action of firms $i = 1, 2$ to remain optimal for every possible action of firm 0.

IV. EQUILIBRIA

■ **Existence.** We are now ready to state our first result.

THEOREM I: An equilibrium exists.

We prove the result by constructing equilibrium strategies and demands. We proceed step by step and by providing intuition. To begin with, we start backwards and try to characterise a strategy for companies $i = 1, 2$. To facilitate our analysis, we first define the following set:

$$\Phi(\alpha_0) = \Big\{\mathbf{x} = (x^\theta)_{\theta\in\Theta} \in \mathbb{X}^{IC} : U^\theta(x^\theta) \geq V^\theta(\alpha_0) \ \forall \ \theta \ \& \ \sum_{\theta\in\Theta} \lambda^\theta \zeta^\theta(x^\theta) \geq 0\Big\}$$

This set includes all allocations that weakly Pareto dominate the payoff profile $\mathbf{V}(\alpha_0)$. Consider now the following symmetric strategy for firms $i = 1, 2$:

$$\bar{\alpha}_j = \begin{cases} \alpha_0, & \text{if} \quad \mathbf{V}(\alpha_0) \in \mathcal{U}^E \\ \beta : \mathbf{V}(\beta) \in \mathcal{U}^E \ \& \ \mathbf{V}(\beta) \geq \mathbf{V}(\alpha_0), & \text{if} \quad \mathbf{V}(\alpha_0) \notin \mathcal{U}^E \ \& \ \Phi(\alpha_0) \neq \emptyset \\ \emptyset & \text{if} \quad \Phi(\alpha_0) = \emptyset \end{cases} \tag{1}$$

The strategy provided in (1) is rather simple. Company i offers α_0, i.e. the same as the market maker, if the market maker has offered a menu of products that is efficient. If the market maker has offered a menu of products that is not efficient, then company i offers a menu of products that (weakly) Pareto dominates the offer of the market maker and is efficient. Lastly, if the market maker has offered a menu of products that makes negative profits, then company i stays out of the market.

Strategy (1) and action $\bar{\alpha}_0$ look like a good starting point towards a characterisation of equilibrium. They need to be complemented by demand functions that satisfy our definition of equilibrium. Unfortunately, it is perhaps impossible to analytically characterise a demand profile. This would require us to analytically describe the demand function for every firm and every possible action, given all possible actions for the rest of the firms. Fortunately, it only suffices to locally characterise a demand profile and to show how, based on that characterisation, a Nash equilibrium is sustained.

Consider, first $\bar{\alpha}_0$ such that $\mathbf{V}(\bar{\alpha}_0) \in \mathcal{U}^E$. If $\alpha_j = \bar{\alpha}_0$ for every i, then $\sum_{x\in\xi^\theta(\bar{\alpha}_0)} \bar{Q}^\theta_0(x, \bar{\boldsymbol{\alpha}}) = \sum_{x\in\xi^\theta(\bar{\alpha}_0)} \bar{Q}^\theta_1(x, \bar{\boldsymbol{\alpha}}) = \sum_{x\in\xi^\theta(\bar{\alpha}_0)} \bar{Q}^\theta_2(x, \bar{\boldsymbol{\alpha}}) = \frac{\lambda^\theta}{3}$ for every θ. In other words, when all firms offer the same menu of products (which is efficient), consumers of every type are uniformly distributed among the three firms such that each firm attracts the same measure of consumers of the

[8] Where $-i$ is the firm other than 0 and i.

same type. If $\alpha_j \neq \bar{\alpha}_0$ and $\alpha_{-j} = \bar{\alpha}_0$, then $\sum_{x \in \xi^\theta(\bar{\alpha}_0)} \bar{Q}^\theta_j(x, \bar{\boldsymbol{\alpha}}) = \lambda^\theta$ for every θ and $\bar{Q}^\theta_0(x, \bar{\boldsymbol{\alpha}}) = \bar{Q}^\theta_{-j}(x, \bar{\boldsymbol{\alpha}}) = 0$ for every $x \in X$ and θ. Verbally, if company j unilaterally deviates and introduces a different menu than the one offered by company 0, i.e. $\bar{\alpha}_0$, it expects to attract all consumers in the market (or at least a sufficiently large number). Recall that all consumers lose nothing switching from firm 0 to firm j since the latter supplies the products that are also supplied by firm 0 (by definition of our mechanism).

Consider now $\tilde{\alpha}_0$ such that $\mathbf{V}(\tilde{\alpha}_0) \in \mathcal{U}^E$. If $\Phi(\tilde{\alpha}_0) = \emptyset$ and $\alpha_1 = \alpha_2 = \emptyset$, then $\sum_{x \in \xi^\theta(\tilde{\alpha}_0)} \bar{Q}^\theta_0(x, \tilde{\boldsymbol{\alpha}}) = \lambda^\theta$ for every θ. In other words, if company 0 offers a menu that makes strictly negative profits, then it attracts all consumers. If $\Phi(\tilde{\alpha}_0) \neq \emptyset$ $\alpha_i \in \Phi(\tilde{\alpha}_0)$ and $\mathbf{V}(\alpha_j) \in \mathcal{U}^E$ for every j, then $\sum_{x \in \xi^\theta(\tilde{\alpha}_1)} \bar{Q}^\theta_1(x, \tilde{\boldsymbol{\alpha}}) = \sum_{x \in \xi^\theta(\tilde{\alpha}_2)} \bar{Q}^\theta_2(x, \tilde{\boldsymbol{\alpha}}) = \frac{\lambda^\theta}{2}$ for every θ and $\bar{Q}^\theta_0(x, \tilde{\boldsymbol{\alpha}}) = 0$ for every θ and $x \in X$.

It is easy to see that $(\bar{\boldsymbol{\alpha}}, \bar{\mathbf{Q}})$, where $\bar{\alpha}_j$ is the symmetric strategy described in (1) and $\bar{\mathbf{Q}}$ is as described (locally at least) in the previous paragraph satisfy our definition of equilibrium. Given the strategies of the two other firms and the demand functions, $\bar{\alpha}_0$ such that $\mathbf{V}(\tilde{\alpha}_0) \in \mathcal{U}^E$ is obviously the best firm 0 can do. Indeed if it deviates to any other menu $\tilde{\alpha}$, then if this menu is such that $\Phi(\tilde{\alpha}) = \emptyset$, the two other firms stay out of the market, company 0 attracts the whole market and makes strictly negative profits. It can make positive profits if it introduces $\bar{\alpha}_0$. If $\tilde{\alpha}$ is such that $\Phi(\tilde{\alpha}) \neq \emptyset$, then, once more, given the strategies of the other two firms and the demand functions, both firms enter the market, introduce a menu that Pareto dominates $\tilde{\alpha}$ and firm 0 will end up with no customers. This action makes zero profits which is (weakly) worse than firm 0 introducing $\bar{\alpha}_0$. Therefore, Condition (i) of our definition of equilibrium is satisfied. With regards to the strategy of firm i,[9] given the action of firm 0, the strategy of firm $-i$, and the demand specification, firm i do its best by playing the strategy provided in (1). To see this, consider a possible deviation by company i. If it offers a menu that is strictly Pareto dominated by $\bar{\alpha}_0$, then firm 0 attracts all consumers in the market and therefore firm i makes zero profits which is (weakly) worse than offering $\bar{\alpha}_0$. If on the other hand, it introduces another menu that attracts some of the types, given $\bar{\alpha}_0$, then the demand is such that, it attracts a sufficient number of all types which renders the deviation unprofitable. Hence, the strategy provided in (1) is sequentially rational.

The discrepancy between our universal existence theorem and the non-existence result of RS could be intuitively explained as follows. RS imagine a market in which an insurance company can freely enter the market and offer a unique insurance policy, disregarding all those policies offered by rivals. As such, an entrant has an incentive to "skim the cream", i.e. attract the relatively low-risk individuals who are less costly, leaving rivals making losses. In our mechanism, every possible entrant has to take into account the minimum standards, as these have been set by the market maker, before entering the market. When the standards are set efficiently, then no firm can attract only high-profitable types without also attracting low-profitable ones.

Note that the characterisation of equilibrium heavily relies on the specification of the demand profile and the fact that there are two firms other than the market maker. As we discuss in the robustness section below, the latter is only for simplicity and to suppress notation. The result goes through, under minor modifications in the mechanism, even when there are two firms. The former

[9]Recall that all firms are symmetric.

is important. In particular, in our characterisation, we have assumed that when a firm other than the market maker tries to "skim the cream" by attracting all profitable types then, because of the minimum standards, it cannot get rid of the most unprofitable types which renders the deviation unprofitable. By foreseeing such a behaviour, no firm has an incentive to play something different than the minimum standards imposed by the market maker. The only intuitive explanation we could give for such a characterisation is that when consumers observe a deviation from some efficient menu, they expect that the rest of the firms in the market would go bankrupt and therefore would be better off switching to the deviant firm. As such, it is prudent to also switch to the firm that sets a different menu of products, besides the minimum standards, even if the terms for their preferred product remains the same among all firms.

□ **Efficiency.** Even though Theorem I proves, under very general conditions, that an equilibrium exists, it does not fully characterise the equilibrium set. For further characterisation, one needs to impose further structure. In what follows, we examine environments that satisfy Assumption A1. We can then state and prove our second main result:

THEOREM II: If Assumption A1 is satisfied, $(\bar{\boldsymbol{\alpha}}, \bar{\mathbf{Q}})$ is a subgame perfect Nash equilibrium if and only if $V(\bar{\boldsymbol{\alpha}}) \in \mathcal{U}^E$.

A formal proof is provided in Appendix A. The proof heavily exploits the strict monotonicity, strict concavity and type-independency of the utility index. Because of these properties, for every inefficient menu of products, we can find another, Pareto improving, menu of products with infinitesimal less (average) profit such that one of the firms prefers to introduce it and conquer the market. As such, no inefficient payoff profile can be sustained as an equilibrium payoff profile.

Note that the same argument does not go through if the utility index is type-dependent, e.g. if consumers differ also with respect to their risk-aversion (three-dimensional heterogeneity), and we insist on deterministic products. In that case, random products need to be considered. A random product is a lottery with support the space of feasible products. The main difficulty with type-dependent utility indexes is that, without the use of random plans, one cannot work with the usual certainty equivalents, as we basically do in the proof of Theorem II, that are incentive compatible and increase profits. The strain lies exactly on the type-dependence of the ex post utility function. Random plans linearise the space of menus of products and make sure that the mix of two incentive compatible (random) allocations is itself incentive compatible.

V. DISCUSSION

■ **Robustness.** The game we analysed in this paper is not unique but there exist different payoff - equivalent games. We used what we believed is the simplest, in which one firm moves first, sets the minimum standards in terms of products, and two other firms simultaneously offer more products on top of the minimum standards. In this formulation, the two firms other than the market market have perfect information about the products set by the market maker. We could instead employ a two-stage game in which in the first stage all firms offer menus of products that will serve as the minimum standards, and in the second stage all companies freely compete by offering more products.

We have also assumed that there are three active firms in the market. That is only for simplicity and it is not important for our results. Note however that, with only two firms, the game needs to be slightly modified. The main difficultly with the current formulation, if there are only two firms, is that strategies and demand functions can be constructed such that once the market maker sets a non-efficient menu of products, the other firm always matches that menu and attracts a sufficient share of consumers rendering therefore any deviation from the market maker unprofitable.[10] Nonetheless, the set of equilibrium indirect utility profiles remains the same as long as we allow the market maker to set the minimum standards in the first stage and compete in the second stage as a "new" player.

Lastly, a point that requires further discussion is the assumption of the symmetry among firms. Our mechanism seems to work when firms are perfectly symmetric and the identity of the market maker makes no difference. Possible asymmetries might include among others different marginal costs, product differentiation or some other source of market power. However, one needs to compare our results with traditional markets in which firms have equal constant marginal costs and compete *à* la Bertrand. In that case, price competition works at its fullest and the equilibrium allocation is efficient. Further work is required in markets with sufficient firm heterogeneity.

□ **Information Requirements of the Mechanism.** Note that if a regulator knew the type space and distribution, he would be able to set the minimum standards himself. In our environment, rather realistically, this is not possible because the regulator lacks the necessary information to optimally set the minimum standards. In particular, the regulator is unaware of the type space and/or the distribution of types in the population. Therefore he needs to rely on the truthful revelation by firms.

That is not to say that the regulator requires absolutely no information. On the contrary, for the mechanism to be implemented, the regulator needs to fully observe the products set by firms. We believe that this is an innocuous assumption. Indeed, consumers need to have access to this type of information in order to make rational choices. If consumers could perfectly access this information, we do not see any particular reason why the regulator should not.

Note that due to Theorem II, the regulator can achieve efficiency in every possible equilibrium of the mechanism. In that respect, our approach differs from the mechanism design approach that requires there be some mechanism and some equilibrium in which the required (efficient) outcome is implemented, and approaches the "*full-implementation mechanisms*" inspired by Maskin (1999). Full- implementation requires that an efficient outcome is implemented in every possible equilibrium of the mechanism. Nonetheless, the advantage of our mechanism compared to those

[10] This is mostly due to the lack of restrictions of how demand is shared among firms when these have symmetric menus of products.

discussed in the full implementation literature is that it is way simpler. Such simple mechanisms have been examined for instance by Varian (1994) and Maskin and Tirole (1999).[11]

□ **Equilibria.** In Section IV, we characterised equilibrium strategies such that the market maker sets the minimum standards efficiently and no other firm has an incentive to introduce new products other than the minimum standards. This does not necessarily mean that these are the only possible equilibrium strategies. It turns out that other equilibrium strategies are possible depending on the environment. As we showed in Dosis (2015), in a game where all firms simultaneously and independently offer menus of products, an equilibrium generically exists if the well-known Rothschild-Stiglitz allocation (RSA) is efficient. In the game we examine, one can show that when the RSA is efficient, then equilibrium strategies exist according to which the market maker offers the null menu of products and the two other firms simply offer the RSA.

As in the original studies of RS and Wilson, the main difficulty arises when the RSA is not efficient. Under these circumstances, efficiency requires cross-subsidisation and hence it leaves the door open to cream skimming. This constitutes the use of restrictions in firms' action spaces inevitable to guarantee that equilibrium exists. Yet, even in that case, there is a natural interpretation for the equilibrium strategies. In particular, one could interpret this type of equilibrium strategies as follows. The market maker is perceived as the leader in the market and the rest of the firms as followers. The market maker sets the minimum standards and the rest of the firms simply follow and have no incentive to introduce new products unless the market maker introduces an inefficient menu in which case some of the firms has an incentive to undercut this and gain a higher market share.

□ **Implementation of the Mechanism.** We believe that our mechanism is easily implementable in practice. For instance, the rather controversial Healthcare Exchange Marketplace (HEM), was an initiation from the Federal government in order to promote competition in the market for health insurance and to give access to health insurance to more individuals. HEMs are regulated, online platforms in which insurance companies and consumers meet and trade insurance plans. HEMs' objective is twofold: First, they accommodate fiercer competition among insurance companies, and, second their use facilitate the provision of federal subsidies. For instance, almost all large private insurance markets are heavily regulated with the authorities keeping a close eye on the set of products offered by insurance companies.[12] For instance, in the US, HEMs provide a first class opportunity towards the efficient regulation of the health insurance market in the US and an ideal way for the implementation of our mechanism. This is because of the transparency it provides as well as its organisation. In such highly regulated platforms, the regulator can easily monitor the products offered by companies and can make sure that the mini-

[11] As Maskin and Sjostrom (2002) quote: ".... In fact, we anticipate that, since so much has now been accomplished toward developing implementation theory at a general level, future efforts are likely to concentrate more on concrete applications of the theory, e.g., to plans (see, for instance, Maskin and Tirole (1999)) or to externalities (see, for instance, Varian (1994)), where special structure will loom large."

[12] Consider the US health insurance market or the Swiss and German health insurance markets.

mum standards are always satisfied.

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APPENDIX A

■ **Proof of Theorem II.** The "if" part follows from Theorem I for every possible market. We can state the following two auxiliary lemmas that also hold in every possible market.

LEMMA A.1: $(\boldsymbol{\alpha}, \mathbf{Q})$ is an equilibrium, only if $\Pi_0(\alpha_0, \boldsymbol{\alpha}_{-0}|Q_0) \geq 0$ and $\Pi_i(\alpha_0, \alpha_i, \alpha_{-i}|Q_i) \geq 0$ for every $i = 1, 2$.

PROOF: In case some of the firms was to make negative profits in equilibrium, it could always select to stay out of the market. Q.E.D.

LEMMA A.2: If $(\boldsymbol{\alpha}, \mathbf{Q})$ is an equilibrium, then there exists $i = 1, 2$ such that

$$\Pi_i(\alpha_0, \alpha_i, \alpha_{-i}|Q_i) \leq \Pi_0(\alpha_0, \boldsymbol{\alpha}_{-0}|Q_0) + \sum_{j=1}^{2} \Pi_j(\alpha_0, \alpha_j, \alpha_{-j}|Q_j)$$

PROOF: The result is an immediate consequence of Lemma A.1 because $\Pi_0(\alpha_0, \boldsymbol{\alpha}_{-0}|Q_0) \geq 0$ and $\Pi_i(\alpha_0, \alpha_i, \alpha_{-i}|Q_i) \geq 0$ for every $i = 1, 2$. Q.E.D.

Now, assume that Assumption A.1 is satisfied. To prove the "only if" part, suppose that $(\boldsymbol{\alpha}, \mathbf{Q})$, where $\mathbf{V}(\boldsymbol{\alpha})\neg \in \mathcal{U}^E$, is an equilibrium. The result is a immediate consequence of Lemma and A.2 and the following two auxiliary lemmas.

LEMMA A.3: For every $(\boldsymbol{\alpha}, \mathbf{Q})$ there exists $\bar{\mathbf{x}} \in \mathbb{X}^{IC}$ such that:

1. $Z(\bar{\mathbf{x}}) \geq \Pi_0(\alpha_0, \boldsymbol{\alpha}_{-0}|Q_0) + \sum_{j=1}^{2} \Pi_j(\alpha_0, \alpha_j, \alpha_{-j}|Q_j)$
2. $U^\theta(\bar{x}^\theta) \geq V^\theta(\alpha_0 \cup \alpha_1 \cup \alpha_2)$

PROOF: Consider some $(\boldsymbol{\alpha}, \mathbf{Q})$. If $\alpha_0 = \alpha_1 = \alpha_2 = \bar{\mathbf{x}} \in \mathbb{X}^{IC}$, then the result is trivial.

Denote as $\alpha_i^\theta \subseteq \alpha_i$ the set of payoff maximising products for type θ from firm i. By definition $U^\theta(y) = V^\theta(\alpha_1 \cup \alpha_2)$ for every θ, i and $y \in \alpha_i(\theta)$. Define $\bar{\mathbf{x}} = (\bar{x}^\theta)_{\theta \in \Theta}$ such that $\bar{x}^\theta = (\bar{x}^\theta_\omega)_{\omega \in \Omega}$, where

$$\bar{x}^\theta_\omega = \frac{1}{\lambda(\theta)} \sum_{i=0}^{2} \sum_{y \in \alpha_i^\theta} Q_i^\theta(y, \boldsymbol{\alpha}) y^\theta_\omega(y) \tag{2}$$

The utility of type θ from $\bar{x}^\theta$ is $U^\theta(\bar{x}^\theta) = \sum_{\omega \in \Omega} p^\theta_\omega v_\omega(\bar{x}^\theta_\omega)$. Because $v_\omega(\cdot)$ is concave and strictly increasing for every ω, by Jensen's inequality, we have that $U^\theta(\bar{x}^\theta) \geq V^\theta(\alpha_1 \cup \alpha_2)$ for every θ. Straightforwardly, from (3)

$$\phi_\omega(\bar{x}^\theta_\omega) = \phi_\omega\Big(\frac{1}{\lambda(\theta)} \sum_{i=0}^{2} \sum_{y \in \alpha_i^\theta} Q_i^\theta(y, \boldsymbol{\alpha}) y^\theta_\omega(y)\Big) \tag{3}$$

and because $\phi_\omega(\cdot)$ is concave for every ω

$$\phi_\omega(\bar{x}^\theta_\omega) = \phi_\omega\Big(\frac{1}{\lambda(\theta)}\sum_{i=0}^{2}\sum_{y\in\alpha_i^\theta} Q_i^\theta(y,\boldsymbol{\alpha})y^\theta_\omega(y)\Big) \geq \frac{1}{\lambda(\theta)}\sum_{i=0}^{2}\sum_{y\in\alpha_i^\theta} Q_i^\theta(y,\boldsymbol{\alpha})\phi_\omega(y^\theta_\omega(y)) \tag{4}$$

Multiplying by p^θ_ω and summing up over ω we have

$$\sum_{\omega\in\Omega} p^\theta_\omega \phi_\omega(\bar{x}^\theta_\omega) \geq \frac{1}{\lambda^\theta}\sum_{\omega\in\Omega} p^\theta_\omega \sum_{i=0}^{2}\sum_{y\in\alpha_i^\theta} Q_i^\theta(y,\boldsymbol{\alpha})\phi_\omega(y^\theta_\omega(y)) \tag{5}$$

The left-hand side of (6) is simply $\zeta^\theta(\bar{x}^\theta)$. Multiplying both sides by λ^θ and summing up over θ,

$$\sum_{\theta\in\theta}\lambda^\theta\zeta^\theta(\bar{x}^\theta) = \sum_{\theta\in\theta}\sum_{\omega\in\Omega} p^\theta_\omega \sum_{i=0}^{2}\sum_{y\in\alpha_i(\theta)} Q_i(\theta,y,\boldsymbol{\alpha})\phi_\omega(y^\theta_\omega(y))$$

The left-hand side is $Z(\bar{\mathbf{x}})$ and the right-hand side $\Pi_0(\alpha_0,\boldsymbol{\alpha}_{-0}|Q_0)+\sum_{j=1}^2\Pi_j(\alpha_0,\alpha_j,\alpha_{-j}|Q_j)$. This proves the result. Q.E.D.

LEMMA A.4: For every $\mathbf{x}\in\mathbb{X}^{IC}$ and $\delta>0$ (small enough) with $Z(\mathbf{x})\geq 0$ and $\mathbf{x}\neg\in\mathbb{X}^E$, there exists $\tilde{\mathbf{x}}\in\mathbb{X}^{IC}$ such that $Z(\tilde{\mathbf{x}})>Z(\mathbf{x})-\delta$ and $U^\theta(\tilde{x}^\theta)>U^\theta(x^\theta)$.

PROOF: Consider $\mathbf{x}\in\mathbb{X}^{IC}$ with $Z(\mathbf{x})>0$ and let $\rho=\max\limits_{\omega,\theta} x^\theta_\omega+\tau$, where $\tau>0$. Define $\tilde{x}^\theta_\omega$ such that $\varepsilon v_\omega(x^\theta_\omega)+(1-\varepsilon)v_\omega(\rho)=v_\omega(\tilde{x}^\theta_\omega)$. Because $v_\omega(\cdot)$ is strictly concave and by Jensen's inequality

$$\tilde{x}^\theta_\omega > \varepsilon x^\theta_\omega+(1-\varepsilon)\rho \tag{6}$$

Because $\phi_\omega(\cdot)$ is strictly increasing, it is true that

$$\phi_\omega(\tilde{x}^\theta_\omega) > \phi_\omega(\varepsilon x^\theta_\omega+(1-\varepsilon)\rho) \tag{7}$$

Given that $\phi_\omega(\cdot)$ is concave

$$\phi_\omega(\tilde{x}^\theta_\omega) \geq \varepsilon\phi_\omega(x^\theta_\omega)+(1-\varepsilon)\phi_\omega(\rho) \tag{8}$$

If we multiply (8) by p^θ_ω and sum over ω, we have:

$$\sum_{\omega\in\Omega} p^\theta_\omega\phi_\omega(\tilde{x}^\theta_\omega) \geq \varepsilon\sum_{\omega\in\Omega} p(\theta,\omega)\phi_\omega(x^\theta_\omega)+(1-\varepsilon)\sum_{\omega\in\Omega} p^\theta_\omega\phi_\omega(\rho) \tag{9}$$

The left-hand side of (9) is simply $\zeta^\theta(\tilde{x}^\theta)$ and the right-hand side $\varepsilon\zeta^\theta(x^\theta)+(1-\varepsilon)\sum_{\omega\in\Omega}\phi_\omega(\rho)$. If we multiply by λ^θ and sum over θ we get $Z(\tilde{\mathbf{x}})\geq\varepsilon Z(\mathbf{x})+(1-\varepsilon)\sum_{\theta\in\Theta}\lambda^\theta\sum_{\omega\in\Omega}p^\theta_\omega\phi_\omega(\rho)$. We can re-write this as $Z(\tilde{\mathbf{x}})\geq Z(\mathbf{x})-\delta$, where $\delta=(1-\varepsilon)[Z(\mathbf{x})+\sum_{\theta\in\Theta}\lambda^\theta\sum_{\omega\in\Omega}p^\theta_\omega\phi_\omega(\rho)]$. Note that because $\rho=\max\limits_{\omega,\theta} x^\theta_\omega-\tau$ by definition, $\sum_{\theta\in\Theta}\lambda^\theta\sum_{\omega\in\Omega}p^\theta_\omega\phi_\omega(\rho)>0$ and $Z(\mathbf{x})>0$ and hence $\delta>0$.

What remains to be shown is that $\tilde{\mathbf{x}} \in \mathbb{X}^{IC}$. By definition $\mathbf{x} \in \mathbb{X}^{IC}$, or $U^\theta(x^\theta) \geq U^\theta(x^{\theta'})$ for every $\theta, \theta' \in \Theta$. Equivalently,

$$\sum_{\omega \in \Omega} p^\theta_\omega v_\omega(x^\theta_\omega) \geq \sum_{\omega \in \Omega} p(^\theta_\omega v_\omega(x^{\theta'}_\omega) \tag{10}$$

If we multiply (10) by ε and add $(1-\varepsilon)\sum_{\omega \in \Omega} p^\theta_\omega v_\omega(\rho)$ in both sides

$$\varepsilon \sum_{\omega \in \Omega} p^\theta_\omega v_\omega(x^\theta_\omega) + (1-\varepsilon)\sum_{\omega \in \Omega} p^\theta_\omega v_\omega(\rho) \geq \varepsilon \sum_{\omega \in \Omega} p^\theta_\omega v_\omega(x^{\theta'}_\omega) + (1-\varepsilon)\sum_{\omega \in \Omega} p^\theta_\omega v_\omega(\rho) \tag{11}$$

Which can be re-written as: $\sum_{\omega \in \Omega} p^\theta_\omega[\varepsilon v_\omega(x^\theta_\omega) + (1-\varepsilon)v_\omega(\rho)] \geq \sum_{\omega \in \Omega} p^\theta_\omega[\varepsilon v_\omega(x^{\theta'}_\omega) + (1-\varepsilon)v_\omega(\rho)]$. Note that the left hand side is $U^\theta(\tilde{x}^\theta)$ and the right-hand side $U^\theta(\tilde{x}^{\theta'})$. Q.E.D.

Due to Lemma A.2, there exists at least one firm $i \in \{1, 2\}$ with a total expected profit less than the aggregate industry profit. Due to Lemmas A.3 and A.4, there exists some menu of products that makes a profit for that firm less than $\delta > 0$ (infinitesimally small) of the aggregate industry profit. That means that this menu of products provides firm i with a profit higher than the equilibrium profit which contradicts the definition of equilibrium. Q.E.D.